**** MLADEN SRAGA ****

2011.

UNIVERZALNA ZBIRKA POTPUNO RIJEŠENIH ZADATAKA

PRIRUČNIK ZA SAMOSTALNO UČENJE

MATEMATIKA

2

EKSPONENCIJALNE i LOGARITAMSKE FUNKCIJE

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$\sqrt{\alpha}$

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Potpunu garanciju na kompletnu zbirku daje: centar za dopisnu poduku M.I.M.-SRAGA - dakle sve što vam se čini nejasno krivo ili sumnjivo - zovite **01-4578-431** ili **01-4579-130** i tražite dodatne upute i objašnjenja ...
Dodatne upute i objašnjenja možete zatražiti i na mail: mim-sraga@zg.htnet.hr

Ovo je jako skraćena varijanta naše zbirke … samo oglednih 40-ak zadataka ….

Ovo nisu svi zadaci iz ove zbirke ,
Ovo je samo manji dio zadataka iz kompletne zbirke …
I ovdje su postavljeni samo kao ogledni primjerci ….
Ali vam mogu poslužiti kao solidna vježba pred testove ili ispitivanja u školi …

1. Izračunaj:

1) $\log_5 25$ 2) $\log_3 3$ 3) $\log_2 8$

4) $\log_8 2$ 5) $1\log_9 3$ 6) $\log_4 0.25$

7) $\log 100$ 8) $1\log 0.1$ 9) $\log 1000$

10) $\log 0.01$

2. Koliko je:

1) $\log_2 8$ 2) $\log_3 27$ 3) $\log_2 16$

4) $\log_{\frac{1}{2}} 16$ 5) $\log_{\frac{1}{2}} 32$ 6) $\log_{\frac{1}{3}} 27$

7) $\log_{\frac{1}{3}} \sqrt{3}$ 8) $\log_{\frac{1}{2}} 64$ 9) $\log_2 \sqrt{2}$

3. Izračunaj:

1) $\log_2 x = 4$ 2) $\log_3 x = 9$ 3) $\log_{\frac{1}{2}} x = 2$

4) $\log_9 x = \frac{1}{2}$ 5) $\log_8 x = \frac{1}{3}$ 6) $\log_3 x = \frac{1}{2}$

7) $\log_2 x = \frac{1}{3}$ 8) $\log_2 x = 0$ 9) $\log_{0.1} x = 2$

10) $\log_{25} x = \frac{1}{2}$ 11) $\log_{27} x = \frac{1}{3}$

4. Izračunaj:

1) $\log_x 9 = \frac{2}{3}$ 2) $\log_x 16 = \frac{4}{5}$ 3) $\log_x 36 = -2$

4) $\log_x 81 = -\frac{4}{3}$ 5) $\log_x \frac{1}{16} = -8$ 6) $\log_x \sqrt{3} = \frac{1}{2}$

7) $\log_x \sqrt[3]{2} = \frac{1}{3}$ 8) $\log_x \sqrt[3]{2} = \frac{1}{6}$ 9) $\log_x 25 = 2$

10) $\log_{25} x = \frac{1}{2}$ 11) $\log_x \frac{8}{27} = \frac{1}{3}$

5.

1) $3^{\log_3 5}$ 2) $2^{\log_2 7}$ 3) $\left(\frac{1}{2}\right)^{\log_2 5}$

4) $3^{\log_9 6}$ 5) $4^{\log_2 5}$ 6) $25^{\log_5 2}$

7) $3^{\log_9 25}$ 8) $\left(\frac{1}{4}\right)^{\log_4 5}$

6. Logaritmiraj:

1) $\log(10x)$ 2) $\log 100x^2$ 3) $\log\left(x^2 y^3 z^4\right)$

4) $\log\frac{10}{x}$ 5) $\log\frac{x^2}{y^3}$ 6) $\log x^2$

7) $\log x^3$ 8) $\log\frac{x^3}{y^5}$ 9) $\log\frac{\sqrt{x}}{y^3}$

10) $\log\sqrt[3]{x}$ 11) $\log\sqrt[5]{x}$ 12) $\log\sqrt[4]{\frac{10}{x}}$

13) $\log\sqrt[3]{100x^2}$ 14) $\log\sqrt[5]{10x^3}$ 15) $\log 1$

16) $\log_3 1$ 17) $\log_5 1$

7. Logaritmiraj

1) $\log\frac{100}{x^2}$ 2) $\log\frac{x^2}{10y^3}$ 3) $\log\frac{\sqrt{xy}}{100}$

4) $\log\frac{\sqrt[3]{x^2}}{100}$ 5) $\log\frac{100x^2}{\sqrt{x}}$ 6) $\log\frac{x^2y^3}{10\sqrt{x}}$

7) $\log\frac{x^2y^3}{\sqrt{10}}$ 8) $\log\sqrt{\frac{10x}{\sqrt{y}}}$ 9) $\log\sqrt[5]{\frac{x^2y^3}{1000}}$

8.

1) $\frac{\log 36}{\log 2+\log 3}$ 2) $\frac{\log 324}{\log 3+\log 6}$ 3) $\frac{\log 3-\log 5}{\log\frac{9}{25}}$

4) $\frac{\log\frac{4}{25}}{\log 2-\log 5}$ 5) $\frac{1+\log 2.5}{\log 35-\log 7}$

9.

1) $\log(x+2)+\log(x-4)=\log(2x-3)$

2) $\log(x+3)+\log(x-1)=\log(5x+7)$

3) $\log(x-2)+\log(x+3)=\log(4x+2)$

4) $\log x+\log(x+2)=\log(7x-6)$

10.

1.) $\frac{3}{\log x+2}=2-\log x$

Ovo nisu svi zadaci iz naše prodajne ZBIRKE o LOGARITAMSKIM I EKSPONENCIJALNIM FUNKCIJAMA već samo jedna manji dio izabranih zadataka ….
kompletnu zbirku u štampanom obliku
možete naručiti na : 01-4578-431 ili preko maila: mim-sraga@zg.htnet.hr

Rješenja zadataka slijede u nastavku:

DETALJNO RIJEŠENI ZADACI PO ŠKOLSKOJ ZBIRCI ZA GIMNAZIJE I TEHNIČKE ŠKOLE (po autorima Dakić-Elezović)
NALAZE SE U OVOM DOKUMENTU od 28 stranice pa na dalje …

Ovdje su i linkovi na još riješenih zadataka po školskoj zbirci:

1. Eksponencijalne i logaritamske funkcije 1.dio
2. Eksponencijalne i logaritamske funkcije 2. dio

Ako trebate kompletno sve riješene zadatke po školskoj zbirci
- kontaktirajte me mailom ili na 098-488-515

Matematika-2- UNIVERZALNA ZBIRKA POTPUNO RIJEŠENIH ZADATAKA
za DRUGI RAZRED SREDNJE ŠKOLE
LOGARITMI KOMPLETNA RJEŠENJA

1.

1) $\log_5 25 = ?$

$\log_5 25 = x$

$25 = 5^x$

$5^2 = 5^x$

$2 = x$

$x = 2$

Zadatak rješavamo pomoću pravila br.1. : $\log_a b = c \Rightarrow b = a^c$

uvdemo nepoznanicu x , pa imamo $a = 5,\ b = 25,\ c = x$

$b = a^c$, $25 = 5^2$

primjenimo pravilo br.30

Pravila za LOGARITMIRANJE

1) $\log_5 25 = x$

$25 = 5^x$

$5^2 = 5^x$

$2 = x$

$x = 2$

2) $\log_3 3 = x$

$3 = 3^x$

$3^1 = 3^x$

$1 = x$

$x = 1$

3) $\log_2 8 = x$

$8 = 2^x$

$2^3 = 2^x$

$3 = x$

$x = 3$

4) $\log_2 8 = x$

$2 = 8^x$

$2^1 = \left(2^3\right)^x$

$2^1 = 2^{3x}$

$1 = 3x$

$3x = 1 \quad / : 3$

$x = \frac{1}{3}$

5) $\log_9 3 = x$

$3 = 9^x$

$3^1 = \left(3^2\right)^x$

$1 = 2x$

$2x = 1 \quad / : 2$

$x = \frac{1}{2}$

6) $\log_4 0.25 = x$

$0.25 = 4^x$

$\frac{25}{100} = 4^x$

$\left(\frac{1}{4}\right) = 4^x$

$4^{-1} = 4^x$

$-1 = x$

$x = -1$

1.

7) $\log 100 = x$

$\log_{10} 100 = x$

$100 = 10^x$

$10^2 = 10^x$

$2 = x$

$x = 2$

8) $\log 0.1 = x$

$\log_{10} 0.1 = x$

$0.1 = 10^x$

$\frac{1}{10} = 10^x$

$10^{-1} = 10^x$

$-1 = x$

$x = -1$

9) $\log 1000 = x$

$\log_{10} 1000 = x$

$1000 = 10^x$

$10^3 = 10^x$

$3 = x$

$x = 3$

10) $\log 0.01 = x$

$\log_{10} 0.01 = x$

$0.01 = 10^x$

$\frac{1}{100} = 10^x$

$\frac{1}{10^2} = 10^x$

$10^{-2} = 10^x$

$-2 = x$

$x = -2$

2. Koliko je:

1) $\log_2 8 = x$

$8 = 2^x$

$2^3 = 2^x$

$3 = x$

$x = 3$

2) $\log_3 27 = x$

$27 = 3^x$

$3^3 = 3^x$

$3 = x$

$x = 3$

3) $\log_2 16 = x$

$16 = 2^x$

$2^4 = 2^x$

$4 = x$

$x = 4$

4) $\log_{\frac{1}{2}} 16 = x$

$16 = \left(\frac{1}{2}\right)^x$

$2^4 = \left(2^{-1}\right)^x$

$2^4 = 2^{-x}$

$4 = -x$

$x = -4$

5) $\log_{\frac{1}{2}} 32 = x$

$32 = \left(\frac{1}{2}\right)^x$

$2^5 = \left(2^{-1}\right)^x$

$2^5 = 2^{-x}$

$5 = -x$

$x = -5$

6) $\log_{\frac{1}{3}} 27 = x$

$27 = \left(\frac{1}{3}\right)^x$

$3^3 = \left(3^{-1}\right)^x$

$3^3 = 3^{-x}$

$3 = -x$

$x = -3$

7) $\log_{\frac{1}{3}} \sqrt{3} = x$

$\sqrt{3} = \left(\frac{1}{3}\right)^x$

$3^{\frac{1}{2}} = \left(3^{-1}\right)^x$

$3^{\frac{1}{2}} = 3^{-x}$

$\frac{1}{2} = -x$

$x = -\frac{1}{2}$

8) $\log_{\frac{1}{2}} 64 = x$

$64 = \left(\frac{1}{2}\right)^x$

$2^6 = \left(2^{-1}\right)^x$

$2^6 = 2^{-x}$

$6 = -x$

$x = -6$

9) $\log_2 \sqrt{2} = x$

$\sqrt{2} = 2^x$

$2^{\frac{1}{2}} = 2^x$

$\frac{1}{2} = x$

$x = \frac{1}{2}$

3.

1) $\log_2 x = 4$

$x = 2^4$

$x = 16$

2) $\log_3 x = 9$

$x = 3^9$

3) $\log_{\frac{1}{2}} x = 2$

$x = \left(\frac{1}{2}\right)^2$

$x = \frac{1}{4}$

4) $\log_9 x = \frac{1}{2}$

$x = (9)^{\frac{1}{2}}$

$x = \sqrt{9}$

$x = 3$

5) $\log_8 x = \frac{1}{3}$

$x = (8)^{\frac{1}{3}}$

$x = \sqrt[3]{8}$

$x = 2$

6) $\log_3 x = \frac{1}{2}$

$x = 3^{\frac{1}{2}}$

$x = \sqrt{3}$

7) $\log_2 x = \frac{1}{3}$

$x = 2^{\frac{1}{3}}$

$x = \sqrt[3]{2}$

8) $\log_2 x = 0$

$x = 2^0$

$x = 1$

9) $\log_{0.1} x = 2$

$x = 0.1^2$

$x = \left(\frac{1}{10}\right)^2$

$x = \frac{1}{100}$

10) $\log_{25} x = \frac{1}{2}$

$x = 25^{\frac{1}{2}}$

$x = \sqrt{25}$

$x = 5$

11) $\log_{27} x = \frac{1}{3}$

$x = 27^{\frac{1}{3}}$

$x = \sqrt[3]{27}$

$x = \sqrt[3]{3^3}$

$x = 3$

4.

1) $\log_x 9 = \frac{2}{3}$

$9 = x^{\frac{2}{3}} \quad /^{\frac{3}{2}}$

$9^{\frac{2}{3}} = x$

$x = 9^{\frac{3}{2}}$

$x = \left(3^2\right)^{\frac{3}{2}}$

$x = 3^{\frac{2}{1}\cdot\frac{3}{2}}$

$x = 3^3$

$x = 27$

2) $\log_x 16 = \frac{4}{5}$

$16 = x^{\frac{4}{5}}$

$16^{\frac{5}{4}} = x$

$x = 16^{\frac{1}{4}\cdot 5}$

$x = \left(\left(2^4\right)^{\frac{1}{4}}\right)^5$

$x = 2^5$

$x = 32$

3) $\log_x 36 = -2$

$36 = x^{-2} \quad /^{-\frac{1}{2}}$

$36^{-\frac{1}{2}} = x$

$x = 36^{-\frac{1}{2}}$

$x = \frac{1}{36^{\frac{1}{2}}}$

$x = \frac{1}{\sqrt{36}}$

$x = \frac{1}{6}$

4) $\log_x 81 = -\frac{4}{3}$

$81 = x^{-\frac{4}{3}} \quad /^{-\frac{3}{4}}$

$81^{-\frac{3}{4}} = x$

$x = 81^{-\frac{3}{4}}$

$x = \left(3^4\right)^{-\frac{3}{4}}$

$x = 3^{\frac{4}{1}\cdot\left(-\frac{3}{4}\right)}$

$x = 3^{-3}$

$x = \frac{1}{3^3}$

$x = \frac{1}{27}$

4.

5) $\log_x \frac{1}{16} = -8$

$\frac{1}{16} = x^{-8} \quad /^{-\frac{1}{8}}$

$\left(\frac{1}{16}\right)^{-\frac{1}{8}} = x$

$x = \left(\frac{1}{16}\right)^{-\frac{1}{8}}$

$x = \left(\frac{16}{1}\right)^{\frac{1}{8}}$

$x = \left(2^4\right)^{\frac{1}{8}}$

$x = 2^{4 \cdot \frac{1}{8}}$

$x = 2^{\frac{1}{2}}$

$x = \sqrt{2}$

6) $\log_x \sqrt{3} = \frac{1}{2}$

$\sqrt{3} = x^{\frac{1}{2}} \quad /^2$

$\sqrt{3^2} = x$

$3 = x$

$x = 3$

7) $\log_x \sqrt[3]{2} = \frac{1}{3}$

$\sqrt[3]{2} = x^{\frac{1}{3}} \quad /^3$

$\sqrt[3]{2^3} = x$

$2 = x$

$x = 2$

8) $\log_x \sqrt[3]{2} = \frac{1}{6}$

$\sqrt[3]{2} = x^{\frac{1}{6}} \quad /^6$

$\sqrt[3]{2^6} = x$

$x = \sqrt[3]{2^6}$

$x = \left(2^6\right)^{\frac{1}{3}}$

$x = 2^{\frac{6}{3}}$

$x = 2^2$

$x = 4$

9) $\log_x 25 = 2$

$25 = x^2 \quad /^{\frac{1}{2}}$

$25^{\frac{1}{2}} = x$

$x = 25^{\frac{1}{2}}$

$x = \sqrt{25}$

$x = 5$

10) $\log_x \frac{8}{27} = 3$

$\frac{8}{27} = x^3$

$\left(\frac{8}{27}\right)^{\frac{1}{3}} = x$

$x = \left(\frac{8}{27}\right)^{\frac{1}{3}}$

$x = \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}}$

$x = \left(\left(\frac{2}{3}\right)^3\right)^{\frac{1}{3}}$

$x = \frac{2}{3}$

5.

1) $3^{\log_3 5} = x \rightarrow$ po pravilu $a^{\log_a x} = x$

2) $2^{\log_2 7} = 7$

3) $\left(\frac{1}{2}\right)^{\log_2 5} = \left(2^{-1}\right)^{\log_2 5} = 2^{-1\log_2 5} = 2^{\log_2 5^{-1}} = 2^{\log_2 \frac{1}{5}} = \frac{1}{5}$

4) $3^{\log_9 6} = 3^{\log_{3^2} 6} = 3^{\frac{1}{2}\log_3 6} = 3^{\log_3 6^{\frac{1}{2}}} = 6^{\frac{1}{2}} = \sqrt{6}$

5) $4^{\log_2 5} = 2^{2\log_2 5} = 2^{\log_2 5^2} = 5^2 = 25$

6) $25^{\log_5 2} = 5^{2\log_5 2} = 5^{\log_5 2^2} = 2^2 = 4$

7) $3^{\log_9 25} = 3^{\log_{3^2} 25} = 3^{\frac{1}{2}\log_3 25} = 3^{\log_3 25^{\frac{1}{2}}} = 25^{\frac{1}{2}} = \sqrt{25} = 5$

8) $\left(\frac{1}{4}\right)^{\log_4 5} = \left(4^{-1}\right)^{\log_4 5} = 4^{\log_4 5^{-1}} = 5^{-1} = \frac{1}{5}$

Pravila za LOGARITMIRANJE

7. Logaritmiraj:

1) $\log\dfrac{100}{x^2} = \log 100 - \log x^2 = \log 10^2 - 2\log x = 2\log 10 - 2\log x =$

$= 2 \cdot 1 - 2\log x = 2 - 2\log x$

2) $\log\dfrac{x^2}{10y^3} = \log x^2 - \log\left(10y^3\right) = 2\log x - \left(\log 10 + \log y^3\right) = 2\log x - \log 10 - 3\log y =$

$= 2\log x - 1 - 3\log x$

3) $\log\dfrac{\sqrt{xy}}{100} = \log\sqrt{xy} - \log 100 = \log\left(\sqrt{x} \cdot \sqrt{y}\right) - \log 10^2 = \log\sqrt{x} + \log\sqrt{y} - 2 \cdot \log 10 =$

$= \log x^{\frac{1}{2}} + \log y^{\frac{1}{2}} - 2 \cdot 1 = \dfrac{1}{2}\log x + \dfrac{1}{2}\log y - 2$

4) $\log\dfrac{\sqrt[3]{x^2}}{100} = \log\sqrt[3]{x^2} - \log 100 = \log\left(x^2\right)^{\frac{1}{3}} - \log 10^2 = \log x^{\frac{2}{3}} - 2\log 10 =$

$= \dfrac{2}{3} \cdot \log x - 2 \cdot 1 = \dfrac{2}{3}\log x - 2$

5) $\log\dfrac{100x^2}{\sqrt{x}} = \log\left(100x^2\right) - \log\sqrt{x} = \log 100 + \log x^2 - \log x^{\frac{1}{2}} =$

$= \log 10^2 + 2\log x - \dfrac{1}{2}\log x = 2\log 10 + \left(2 - \dfrac{1}{2}\right)\log x =$

$= 2 + \dfrac{3}{2}\log x$

7.

6) $\log\dfrac{x^2y^3}{10\sqrt{x}} = \log\left(x^2y^3\right) - \log\left(10\sqrt{x}\right) = \log x^2 + \log y^3 - \left(\log 10 + \log\sqrt{x}\right) =$

$= 2\log x + x + 3\log y - \log 10 - \log x^{\frac{1}{2}} = 2\log x + 3\log y - 1 - \dfrac{1}{2}\log x$

7) $\log\dfrac{x^2y^3}{\sqrt{10}} = \log\left(x^2y^3\right) - \log\sqrt{10} = \log x^2 + \log y^3 - \log 10^{\frac{1}{2}} =$

$= 2\log x + 3\log y - \dfrac{1}{2}\log 10 = 2\log x + 3\log y - \dfrac{1}{2}$

8) $\log\sqrt{\dfrac{10x}{\sqrt{y}}} = \log\dfrac{\sqrt{10x}}{\sqrt{\sqrt{y}}} = \log\dfrac{\sqrt{10}\cdot\sqrt{x}}{\sqrt[4]{y}} = \log\left(\sqrt{10}\cdot\sqrt{x}\right) - \log\sqrt[4]{y} =$

$= \log\sqrt{10} + \log\sqrt{x} - \log y^{\frac{1}{4}} = \log 10^{\frac{1}{2}} + \log x^{\frac{1}{2}} - \dfrac{1}{4}\log y =$

$= \dfrac{1}{2}\log 10 + \dfrac{1}{2}\log x - \dfrac{1}{4}\log y = \dfrac{1}{2} + \dfrac{1}{2}\log x - \dfrac{1}{4\log y}$

9) $\log\sqrt[5]{\dfrac{x^2y^3}{1000}} = \log\left(\dfrac{x^2y^3}{1000}\right)^{\frac{1}{5}} = \dfrac{1}{5}\log\dfrac{x^2y^3}{1000} = \dfrac{1}{5}\left(\log\left(x^2y^3\right) - \log 1000\right) =$

$= \dfrac{1}{5}\left(\log x^2 + \log y^3 - \log 10^3\right) = \dfrac{1}{5}\left(2\log x + 3\log y - 3\log 10\right) =$

$= \dfrac{1}{5}\cdot 2\log x + \dfrac{1}{5}\cdot 3\log y - \dfrac{1}{5}\cdot 3\cdot 1 = \dfrac{2}{5}\log x + \dfrac{3}{5}\log y - \dfrac{3}{5}$

6. Logaritmiraj:

1) $\log(10x) = \log 10 + \log x = 1 + \log x$

2) $\log 100x^2 = \log 100 + \log x^2 = \log 10^2 + 2\log x = 2\log 10 + 2\log x =$
$= 2 \cdot 1 + 2\log x = 2 + 2\log x$

3) $\log\left(x^2 y^3 z^4\right) = \log x^2 + \log y^3 + \log z^4 = 2\log x + 3\log y + 4\log z$

4) $\log \frac{10}{x} = \log 10 - \log x = 1 - \log x$

5) $\log \frac{x^2}{y^3} = \log x^2 - \log y^3 = 2\log x - 3\log y$

6) $\log x^2 = 2\log x$

7) $\log x^3 = 3\log x$

8) $\log \frac{x^3}{y^5} = \log x^3 - \log y^5 = 3\log x - 5\log y$

9) $\log \frac{\sqrt{x}}{y^3} = \log \sqrt{x} - \log y^3 = \log x^{\frac{1}{2}} - 3\log y = \frac{1}{2}\log x - 3\log y$

10) $\log \sqrt[3]{x} = \log x^{\frac{1}{3}} = \frac{1}{3}\log x$

6.

11) $\log\sqrt[5]{x} = \log x^{\frac{1}{5}} = \frac{1}{5}\log x$

12) $\log\sqrt[4]{\frac{10}{x}} = \log\frac{\sqrt[4]{10}}{\sqrt[4]{x}} = \log\sqrt[4]{10} - \log\sqrt[4]{x} = \log 10^{\frac{1}{4}} - \log x^{\frac{1}{4}} = \frac{1}{4}\log 10 - \frac{1}{4}\log x =$

$= \frac{1}{4}\cdot 1 - \frac{1}{4}\log x = \frac{1}{4} - \frac{1}{4}\log x$

13) $\log\sqrt[3]{100x^2} = \log\left(100x^2\right)^{\frac{1}{3}} = \frac{1}{3}\log\left(100\cdot x^2\right) = \frac{1}{3}\left(\log 100 + \log x^2\right) =$

$= \frac{1}{3}\cdot\left(\log 10^2 + 2\log x\right) = \frac{1}{3}\cdot\left(2\log 10 + 2\log x\right) = \frac{1}{3}\left(2\cdot 1 + 2\log x\right) =$

$= \frac{2}{3} + \frac{2}{3}\log x$

14) $\log\sqrt[5]{10x^3} = \log\left(10x^3\right)^{\frac{1}{5}} = \frac{1}{5}\log\left(10x^3\right) = \frac{1}{5}\left(\log 10 + \log x^3\right) =$

$= \frac{1}{5}\left(1 + 3\log x\right) = \frac{1}{5} + \frac{3}{5}\log x$

15) $\log 1 = 0 \rightarrow$ po pravilu $\log_a 1 = 0$

16) $\log_3 1 = 0$

17) $\log_5 1 = 0$

8.

1) $$\frac{\log 36}{\log 2+\log 3}=\frac{\log 6^2}{\log(2\cdot 3)}=\frac{2\log 6}{\log 6}=\frac{2\cancel{\log 6}}{\cancel{\log 6}}=2$$

2) $$\frac{\log 324}{\log 3+\log 6}=\frac{\log 18^2}{\log(3\cdot 6)}=\frac{2\log 18}{\log 18}=\frac{2\cancel{\log 18}}{\cancel{\log 18}}=2$$

3) $$\frac{\log 3-\log 5}{\log\frac{9}{25}}=\frac{\log\frac{3}{5}}{\log\left(\frac{3}{5}\right)^2}=\frac{\log\frac{3}{5}}{2\log\frac{3}{5}}=\frac{1\cdot\cancel{\log\frac{3}{5}}}{2\cdot\cancel{\log\frac{3}{5}}}=\frac{1}{2}$$

4) $$\frac{\log\frac{4}{25}}{\log 2-\log 5}=\frac{\log\left(\frac{2}{5}\right)^2}{\log\frac{2}{5}}=\frac{2\cdot\log\frac{2}{5}}{\log\frac{2}{5}}=\frac{2\cdot\cancel{\log\frac{2}{5}}}{\cancel{\log\frac{2}{5}}}=2$$

5) $$\frac{1+\log 2.5}{\log 35-\log 7}=\frac{\log 10+\log 2.5}{\log\frac{35}{7}}=$$

$$=\frac{\log(10\cdot 2.5)}{\log 5}=\frac{\log 25}{\log 5}=\frac{\log 5^2}{\log 5}=\frac{2\log 5}{\log 5}=\frac{2\cancel{\log 5}}{\cancel{\log 5}}=2$$

Pravila za LOGARITMIRANJE

9.

1) $\log(x+2)+\log(x-4)=\log(2x-3)$

uvjeti :

$x+2>0 \quad x-4>0 \quad 2x-3>0$

$x>-2 \quad x>4 \quad 2x>3$

$x>\frac{3}{2}$

$\Downarrow$

$x>4$

$\log\left[(x+2)(x-4)\right]=\log(2x-3)$

$(x+2)(x-4)=2x-3$

$x^2-4x+2x-8-2x+3=0$

$x^2-4x-5=0$

$$x_{1,2}=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot(-5)}}{2\cdot 1}=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm\sqrt{36}}{2}=\frac{4\pm 6}{2}$$

$$x_1=\frac{4-6}{2}=-\frac{2}{2} \qquad x_2=\frac{4+6}{2}=\frac{10}{2}$$

$x_1=-1 \qquad x_2=5$

uvjet kaže da mora biti: $x>4$ pa je jedino rješenje $x=5$

2) $\log(x+3)+\log(x-1)=\log(5x+7)$

uvjeti:

$x+3>0 \quad x-1>0 \quad 5x+7>0$

$x>-3 \quad x>1 \quad 5x>-7$

$x>-\frac{7}{5}$

$\Downarrow$

uvijet je $x>1$

$\log\left[(x+3)\cdot(x-1)\right]=\log(5x+7)$

$(x+3)\cdot(x-1)=5x+7$

$x^2-x+3x-3-5x-7=0$

$x^2-3x-10=0$

$$x_{1,2}=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 1\cdot(-10)}}{2\cdot 1}=\frac{3\pm\sqrt{9+40}}{2}=\frac{3\pm\sqrt{49}}{2}=\frac{3\pm 7}{2}$$

$$x_1=\frac{3-7}{2}=-\frac{4}{2} \qquad x_2=\frac{3+7}{2}=\frac{10}{2}$$

$x_1=-2 \qquad x_2=5$

uvjet kaže da mora biti: $x>1$

$\Downarrow$

pa je: $x=5$ jedino rješenje

3) $\log(x-2)+\log(x+3)=\log(4x+2)$

4) $\log x+\log(x+2)=\log(7x-6)$

10.

1.) $\dfrac{3}{\log x+2}=2-\log x$

uvjet:

$\log x+2\neq 0$

$\log x\neq -2$

$x\neq 10^{-2}$

$x\neq \dfrac{1}{100}$

$\log x=t$

$\dfrac{3}{t+2}=2-t \quad /\cdot(t+2)$

$3=(2-t)(t+2)$

$3=2^2-t^2$

$3=4-t^2$

$3-4=-t^2$

$-1=-t^2$

$t^2=1 \quad /\sqrt{\ }$

$t=\pm\sqrt{1}$

$t_1=-1 \qquad t_2=1$

$\log x=t$

$\log x=-1$	$\log x=1$
$x=10^{-1}$	$x=10^{1}$
$x=\dfrac{1}{10}$	$x=10$

uvjet: $x\neq\dfrac{1}{100}$

pa su rješenja: $x_1=\dfrac{1}{10}$ i $x_2=10$

Ovo su ogledni primjeri stranica iz
ZBIRKE POTPUNO RIJEŠENIH ZADATAKA
EKSPONENCIJALNE I LOGARITAMSKE FUNKCIJE
Za DRUGI razred svih srednjih škola
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Autor: Mladen Sraga
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Zadaci 5.3. riješeni zadaci po školskoj zbirci Dakić-Elezović - 2014.-15.izdanje

9. koristimo pravilo: $\log_a b = c \rightarrow b = a^c$

1.) $\log_x 8 = -\frac{3}{4}$

$x^{-\frac{3}{4}} = 8 \quad / \; -\frac{4}{3}$

$x = 8^{-\frac{4}{3}}$

$x = (2^3)^{-\frac{4}{3}}$

$x = 2^{3 \cdot (-\frac{4}{3})}$

$x = 2^{-4}$

$x = \frac{1}{2^4}$

$x = \frac{1}{16}$

kako se vidi iz zadatka odredimo a,b,c i samo postavimo $a^c = b$

jednadžbu potenciramo RECIPROČNOM vrijednosti potencije na nepoznanici x

u ovom zadatku potencija na x je $-\frac{3}{4}$

a njena recipročna vrijednost je $-\frac{4}{3}$

jer sada se potencija sa x pomnoži sa potencijom kojom potenciramo jednadžbu . . .

i $-\frac{3}{4} \cdot \left(-\frac{4}{3}\right) = 1$ uvijek dobijemo 1

pa nam tako nepoznanica x ima eksp. $= x^1$

pa je ljeva strana jednadžbe gotova , desno je obično potenciranje poznate baze sa nekim eksponentom po već poznatim pravilima . . .

2.) $\log_x \frac{1}{8} = -\frac{3}{2}$

$x^{-\frac{3}{2}} = \frac{1}{8} \quad / \; -\frac{2}{3}$

$x = \left(\frac{1}{8}\right)^{-\frac{2}{3}}$

$x = \left(\frac{1}{2^3}\right)^{-\frac{2}{3}}$

$x = (2^{-3})^{-\frac{2}{3}}$

$x = 2^{-3 \cdot (-\frac{2}{3})}$

$x = 2^2$

$x = 4$

3) $\log_x 64 = -3$

$x^{-3} = 64 \quad / \; -\frac{1}{3}$

$x = 64^{-\frac{1}{3}}$

$x = (2^6)^{-\frac{1}{3}}$

$x = 2^{-\frac{6}{3}}$

$x = 2^{-2}$

$x = \frac{1}{2^2}$

$x = \frac{1}{4}$

4) $\log_x 8 = 1{,}5$

$x^{1,5} = 8$

$x^{\frac{3}{2}} = 2^3 \quad / \; \frac{2}{3}$

$x = (2^3)^{\frac{2}{3}}$

$x = 2^{3 \cdot \frac{2}{3}}$

$x = 2^2$

$x = 4$

Autor zadataka: Mladen Sraga

9.

5) $\log_x \frac{1}{16} = -4$

$x^{-4} = \frac{1}{16} \quad / ^{-\frac{1}{4}}$

$x = \left(\frac{1}{16}\right)^{-\frac{1}{4}}$

$x = \left(\frac{1}{2^4}\right)^{-\frac{1}{4}}$

$x = \left(2^{-4}\right)^{-\frac{1}{4}}$

$x = 2^{-4 \cdot \left(-\frac{1}{4}\right)}$

$x = 2$

$x = 2^1$

$x = 2$

6) $\log_x \sqrt{2} = \frac{1}{4}$

$x^{\frac{1}{4}} = \sqrt{2} \quad / ^4$

$x = \sqrt{2}^{\,4}$

$x = \left(2^{\frac{1}{2}}\right)^4$

$x = 2^{\frac{4}{2}}$

$x = 2^2$

$x = 4$

7) $\log_x 27 = -\frac{3}{4}$

$x^{-\frac{3}{4}} = 27 \quad / ^{-\frac{4}{3}}$

$x = 27^{-\frac{4}{3}}$

$x = \left(3^3\right)^{-\frac{4}{3}}$

$x = 3^{3 \cdot \left(-\frac{4}{3}\right)}$

$x = 3^{-4}$

$x = \frac{1}{3^4}$

$x = \frac{1}{81}$

8) $\log_x 0{,}125 = -2$

$x^{-2} = 0{,}125 \quad / ^{-\frac{1}{2}}$

$x = \left(\frac{125}{1000}\right)^{-\frac{1}{2}}$

$x = \left(\frac{1}{8}\right)^{-\frac{1}{2}}$

$x = 8^{\frac{1}{2}}$

$x = \sqrt{8}$

$x = \sqrt{4 \cdot 2}$

$x = \sqrt{4} \cdot \sqrt{2}$

$x = 2\sqrt{2}$

9) $\log_x \frac{7}{3} = 1$

$\frac{7}{3} = x^1$

$x = \frac{7}{3}$

10) $\log^x \frac{8}{27} = -3$

$\frac{8}{27} = x^{-3} \quad / ^{-\frac{1}{3}}$

$\left(\frac{8}{27}\right)^{-\frac{1}{3}} = x$

$x = \left(\frac{2^3}{3^3}\right)^{-\frac{1}{3}} = \left(\left(\frac{2}{3}\right)^3\right)^{-\frac{1}{3}} = \left(\frac{2}{3}\right)^{3 \cdot \left(-\frac{1}{3}\right)} = \left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

10.

koristimo pravilo: $a^{\log_a x} = x$

1) $5^{\log_5 10} = 10$

2) $\left(\frac{1}{5}\right)^{\log_5 10} = \left(5^{-1}\right)^{\log_5 10} = 5^{-1\log_5 10} = 5^{\log_5 10^{-1}} = 10^{-1} = \frac{1}{10}$

3) $3^{\log_3 3} = 3$

4) $\left(\frac{1}{3}\right)^{\log_3 11} = \left(3^{-1}\right)^{\log_3 11} = 3^{-1\log_3 11} = 3^{\log_3 11^{-1}} = 11^{-1} = \frac{1}{11}$

5) $3^{2\log_9 12} = 3^{\log_9 12^2} = 3^{\log_{3^2} 12^2} = 3^{\frac{1}{2}\log_3 12^2} = 3^{\log_3 (12^2)^{1/2}} =$

$= (12^2)^{\frac{1}{2}} = \sqrt{12^2} = 12$

6) $2^{2\log_4 7} = 2^{\log_4 7^2} = 2^{\log_{2^2} 7^2} = 2^{\frac{1}{2}\log_2 7^2} = 2^{\log_2 (7^2)^{\frac{1}{2}}} =$

$= (7^2)^{\frac{1}{2}} = \sqrt{7^2} = 7$

7) $4^{\log_2 3} = 2^{2\log_2 3} = 2^{\log_2 3^2} = 3^2 = 9$

8) $3^{-2\log_9 20} = 3^{\log_9 20^{-2}} = 3^{\log_{3^2} 20^{-2}} = 3^{\frac{1}{2}\log_3 20^{-2}} = 3^{\log_3 (20^{-2})^{\frac{1}{2}}} =$

$= (20^{-2})^{\frac{1}{2}} = \sqrt{20^{-2}} = \sqrt{\left(\frac{1}{20}\right)^2} = \frac{1}{20}$

11.

1) $100^{-\log\frac{5}{2}} = 100^{\log\left(\frac{5}{2}\right)^{-1}} = (10^2)^{\log\left(\frac{5}{2}\right)^{-1}} = 10^{2\log\left(\frac{5}{2}\right)^{-1}} =$

$= 10^{\log\left[\left(\frac{5}{2}\right)^{-1}\right]^2} = 10^{\log\left(\frac{5}{2}\right)^{-2}} = \left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

11.

2) $81^{\frac{1}{2}\log_3 7} = 81^{\log_3 (7)^{\frac{1}{2}}} = (3^4)^{\log_3 (7)^{\frac{1}{2}}} = 3^{4\log_3 (7)^{\frac{1}{2}}} =$

$= 3^{\log_3 \left[7^{\frac{1}{2}}\right]^4} = \left[7^{\frac{1}{2}}\right]^4 = 7^{\frac{4}{2}} = 7^2 = 49$

3) $8^{-\log_4 9} = 8^{\log_4 9^{-1}} = (2^3)^{\log_4 9^{-1}} = 2^{3\log_{2^2} 9^{-1}} =$

$= 2^{\log_{2^2} (9^{-1})^3} = 2^{\frac{1}{2}\log_2 (9^{-1})^3} = 2^{\log_2 \left[(9^{-1})^3\right]^{\frac{1}{2}}} = \left[(9^{-1})^3\right]^{\frac{1}{2}} =$

$= (9^{-3})^{\frac{1}{2}} = \left[\left(\frac{1}{9}\right)^3\right]^{\frac{1}{2}} = \sqrt{\left(\frac{1}{9}\right)^3} = \sqrt{\frac{1}{729}} = \frac{\sqrt{1}}{\sqrt{729}} = \frac{1}{27}$

4) $9^{-\log_3 8} = 9^{\log_3 8^{-1}} = (3^2)^{\log_3 8^{-1}} = 3^{2\log_3 8^{-1}} = 3^{\log_3 (8^{-1})^2} =$

$= (8^{-1})^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

5) $\left(\frac{1}{2}\right)^{2\log_2 \frac{1}{2}} = \left(\frac{1}{2}\right)^{\log_2 \left(\frac{1}{2}\right)^2} = (2^{-1})^{\log_2 \frac{1}{4}} = 2^{-1\log_2 \frac{1}{4}} = 2^{\log_2 \left(\frac{1}{4}\right)^{-1}} =$

$= \left(\frac{1}{4}\right)^{-1} = 4$

6) $\left(\frac{1}{4}\right)^{-2\log_8 125} = \left(\frac{1}{4}\right)^{\log_8 (125)^{-2}} = \left(4^{-1}\right)^{\log_{2^3} (125^{-2})} = 4^{-1\log_{2^3} (125^{-2})} =$

$= 4^{\log_{2^3} (125^{-2})^{-1}} = 4^{\frac{1}{3}\log_2 125^2} = 4^{\log_2 (125^2)^{\frac{1}{3}}} = (2^2)^{\log_2 (125^2)^{\frac{1}{3}}} =$

$= 2^{2\log_2 \left(125^{\frac{2}{3}}\right)} = 2^{\log_2 \left(125^{\frac{2}{3}}\right)^2} = \left(125^{\frac{2}{3}}\right)^2 = 125^{\frac{4}{3}} = \sqrt[3]{125^4} =$

$= \sqrt[3]{125^3 \cdot 125} = 125\sqrt[3]{125} = 125 \cdot 5 = 625$

12.

$f(x) = \log_5 x$

$f(1) = \log_5 1 = 1$

$f(-2) = \log_5 (-2)$ nije definirano

$f(0.2) = \log_5 0.2 = \log_5 \frac{2}{10}^{1} = \log_5 \frac{1}{5} = \log_5 5^{-1} = -1$

$f(125) = \log_5 125 = \log_5 5^3 = 3$

$f(0.04) = \log_5 0.04 = \log_5 \frac{4}{100}^{1} = \log_5 \frac{1}{25} = \log_5 25^{-1} =$

$= \log_5 (5^2)^{-1} = \log_5 5^{-2} = -2$

13.

$f(x) = \log_{\frac{1}{4}} x$

$f(2) = \log_{\frac{1}{4}} 2 = \log_{4^{-1}} 2 = -\frac{1}{1}\log_4 2 = \log_4 2^{-1} = \log_{2^2} 2^{-1} =$

$= \frac{1}{2}\log_2 2^{-1} = \log_2 (2^{-1})^{\frac{1}{2}} = \log_2 2^{-\frac{1}{2}} = -\frac{1}{2}$

$f(-4) = \log_{\frac{1}{4}} (-4)$ nije definirano

$f(0.25) = \log_{\frac{1}{4}} 0.25 = \log_{\frac{1}{4}} \frac{25}{100}^{1} = \log_{\frac{1}{4}} \frac{1}{4} = 1$

$f(0) = \log_{\frac{1}{4}} 0$ nije definirano

$f(0.5) = \log_{\frac{1}{4}} 0.5 = \log_{\frac{1}{4}} \frac{5}{10}^{1} = \log_{\frac{1}{4}} \frac{1}{2} = \log_{4^{-1}} \frac{1}{2} = -\frac{1}{1}\log_4 \frac{1}{2} =$

$= \log_4 \left(\frac{1}{2}\right)^{-1} = \log_{2^2} \left(\frac{1}{2}\right)^{-1} = \frac{1}{2}\log_2 \left(\frac{1}{2}\right)^{-1} = \log_2 \left[\left(\frac{1}{2}\right)^{-1}\right]^{\frac{1}{2}} =$

$= \log_2 \left(\frac{1}{2}\right)^{-\frac{1}{2}} = \log_2 (2^{-1})^{-\frac{1}{2}} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$

14.

$f(x) = \log x$

$f(0.1) = \log 0.1 = \log \frac{1}{10} = \log 10^{-1} = -1$

$f(100) = \log 100 = \log 10^2 = 2$

$f(0.001) = \log 0.001 = \log \frac{1}{1000} = \log 1000^{-1} = \log (10^3)^{-1} =$

$= \log 10^{-3} = -3$

$f(10^{-5}) = \log 10^{-5} = -5$

14. (nastavak)

$$f(0.01^{-4}) = \log 0.01^{-4} = \log\left(\frac{1}{100}\right)^{-4} = \log (100^{-1})^{-4} =$$

$$= \log 100^4 = \log (10^2)^4 = \log 10^8 = 8$$

15.

$$\lfloor \log 123 \rfloor = \lfloor 2,089 \rfloor = 2$$

$$\lfloor \log 5.5 \rfloor = \lfloor 0,740 \rfloor = 0$$

$$\lfloor \log 0,989 \rfloor = \lfloor -4.80 \cdot 10^{-3} \rfloor = \lfloor -0,0048 \rfloor = -1$$

$$\lfloor \log 0.01 \rfloor = \lfloor -2 \rfloor = -2$$

16.

$$\lfloor \log_2 77 \rfloor = \left\lfloor \frac{\log 77}{\log 2} \right\rfloor = \left\lfloor \frac{1,88649}{0,30103} \right\rfloor = \lfloor 6,266 \rfloor = 6$$

$$\lfloor \log_3 0.1 \rfloor = \left\lfloor \frac{\log 0,1}{\log 3} \right\rfloor = \left\lfloor \frac{-1}{0.47712} \right\rfloor = \lfloor -2.095 \rfloor = -3$$

$$\lfloor \log_8 11111 \rfloor = \left\lfloor \frac{\log 11111}{\log 8} \right\rfloor = \left\lfloor \frac{4,04575}{0.90309} \right\rfloor = \lfloor 4,479 \rfloor = 4$$

$$\lfloor \log_{\frac{1}{4}} 25 \rfloor = \left\lfloor \frac{\log 25}{\log \frac{1}{4}} \right\rfloor = \left\lfloor \frac{1.39794}{-0,60206} \right\rfloor = \lfloor -2.321 \rfloor = -3$$

$$\lfloor \log_{\frac{1}{5}} 0.01 \rfloor = \left\lfloor \frac{\log 0,01}{\log \frac{1}{5}} \right\rfloor = \left\lfloor \frac{-2}{-0,69897} \right\rfloor = \lfloor 2.861 \rfloor = 2$$

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